

Lesson 16. The Points-After-Touchdown Problem

1 The problem

- In an NFL football game, after scoring a touchdown, a team is given the option to try for:
 - a **1-point conversion**: 1 extra point by a field goal from the 15-yard line, or
 - a **2-point conversion**: 2 extra points by advancing the ball into the end zone from the 2-yard line
- Whether to “go for 2” is a classic debate – a few discussions on the topic:
 - <https://theringer.com/nfl-two-point-conversions-pittsburgh-steelers-mike-tomlin-65d47282d853>
 - <https://fivethirtyeight.com/features/more-nfl-teams-are-going-for-two-just-as-they-should-be/>
- Adding to the debate: in 2015, 1-point attempts were moved from the 2-yard line to the 15-yard line
- Conversion success rates from the 2014-2017 regular seasons (from <http://www.pro-football-reference.com/>):

	2014	2015	2016	2017
1-point conversion success rate	0.993	0.942	0.936	0.940
2-point conversion success rate	0.483	0.479	0.486	0.451

- Based on the current score and time remaining, should a team “go for 1” or “go for 2” in order to maximize the probability that it wins the game?
- How does the 2015 rule change affect a team’s optimal conversion strategy?
- Let’s try to answer these questions by modeling this problem as a stochastic dynamic program
- We will be roughly following this paper:

H. Sackrowitz (2000). Refining the point(s)-after-touchdown decision. *Chance* 13(3): 29-34.

2 Data

- Two teams: A and B
 - Assume that we (the decision-makers) are Team A
- Suppose we have the following data:

T = total number of possessions

$p_n = \Pr\{1\text{-pt. conv. successful for Team } n \mid 1\text{-pt. conv. attempted by Team } n\}$ for $n = A, B$

$q_n = \Pr\{2\text{-pt. conv. successful for Team } n \mid 2\text{-pt. conv. attempted by Team } n\}$ for $n = A, B$

$b_1 = \Pr\{1\text{-pt. conv. attempted by Team B}\}$

$b_2 = \Pr\{2\text{-pt. conv. attempted by Team B}\}$

$t_n = \Pr\{\text{TD by Team } n \text{ in 1 possession}\}$ for $n = A, B$

$g_n = \Pr\{\text{FG by Team } n \text{ in 1 possession}\}$ for $n = A, B$

$z_n = \Pr\{\text{no score by Team } n \text{ in 1 possession}\}$ for $n = A, B$

$r = \Pr\{\text{Team A wins in overtime}\}$

- What is the relationship between b_1 and b_2 ?

$$b_1 + b_2 = 1$$

- What is the relationship between t_n , g_n and z_n ?

$$t_n + g_n + z_n = 1$$

- What is the probability that Team B scores 0 after a touchdown?

$$1 - (b_1 p_B + b_2 q_B)$$

Team B goes for 1? Team B makes 1?
Team B goes for 2? Team B makes 2?

3 The stochastic DP

- Stages:

$$\begin{aligned}
 t = 0, 1, \dots, T - 1 &\leftrightarrow \text{end of possession } t \\
 t = T &\leftrightarrow \text{end of game}
 \end{aligned}$$

- For our purposes, a possession ends when a team scores (TD or FG), or loses possession without scoring

- States:

$$\begin{aligned}
 (n, k, d) &\leftrightarrow \text{Team } n\text{'s possession just ended} && \text{for } n \in \{A, B\} \\
 &\text{Team } n \text{ just scored } k \text{ points} && \text{for } k \in \{0, 3, 6\} \\
 &\text{Team A is ahead by } d \text{ points} && \text{for } d \in \{\dots, -1, 0, 1, \dots\}
 \end{aligned}$$

- Value-to-go function:

$$\begin{aligned}
 f_t(n, k, d) = \text{maximum probability that Team A wins when in state } (n, k, d) \text{ at the end of possession } t \\
 \text{for } n \in \{A, B\}, k \in \{0, 3, 6\}, d \in \{\dots, -1, 0, 1, \dots\}
 \end{aligned}$$

- Allowable decisions x_t at stage t and state (n, k, d) :

$$\begin{aligned}
 x_t \in \{1, 2\} &\text{ if } n = A \text{ and } k = 6 \\
 x_t = \text{none} &\text{ if } n = A \text{ and } k \in \{0, 3\} \\
 x_t = \text{none} &\text{ if } n = B \text{ and } k \in \{0, 3, 6\}
 \end{aligned}$$

- We need to consider transitions from the following states:

$$\begin{aligned}
 (A, 6, d) \quad (A, 3, d) \quad (A, 0, d) \\
 (B, 6, d) \quad (B, 3, d) \quad (B, 0, d)
 \end{aligned}
 \quad \text{for all } d$$

- Since our objective is to maximize the probability of winning, we set all the contributions in stages $t = 0, 1, \dots, T - 1$ to 0, just like in the investment problem in Lesson 15











